Exercise 10

Find the particular solution for each of the following initial value problems:

$$u' - 3u = 4x^3e^{3x}, \quad u(0) = 1$$

Solution

This is an inhomogeneous first order linear ODE, so we can multiply both sides by the integrating factor,

$$I(x) = e^{\int (-3)\,dx} = e^{-3x},$$

to solve it. The equation becomes

$$e^{-3x}u' - 3e^{-3x}u = 4x^3.$$

Observe that the left side can be written as $(e^{-3x}u)'$ by the product rule.

$$\frac{d}{dx}(e^{-3x}u) = 4x^3$$

Now integrate both sides with respect to x.

$$e^{-3x}u = x^4 + C$$

The general solution is thus

$$u(x) = e^{3x}(x^4 + C).$$

Because an initial condition is given, this constant of integration can be determined.

$$u(0) = e^0(0 + C) = C \quad \rightarrow \quad C = 1$$

Therefore,

$$u(x) = e^{3x}(x^4 + 1).$$